Rule Interpolation by Spatial Geometric Representation

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Abstract

In this article we will present a new multidimensional fuzzy interpolation method. This method, compared to the existing methods, does not require convex and normal fuzzy sets in the rules, but can be applied for arbitrary type of fuzzy sets. The new method gives an interpretable conclusion in every case, unlike the previously published methods. In the article we will also show a specialized, simplified version of the new method, which uses three of the most wide spread set types in practice: the crisp, the triangular, and the trapezoidal fuzzy sets. The difference between the new and the former methods will be pointed out.

1 INTRODUCTION

The advantages of fuzzy controllers in practice, and the growing number of applications are well known. The basic job of fuzzy controllers is to transform the input signal into membership degrees (observation fuzzy set), then to generate conclusion fuzzy set and, depending on needs, to deffuzify the conclusion [1]. Naturally a conclusion to an observation can be only given on the base of the knowledge got from earlier information, so a conclusion set can only be generated starting from the fuzzy sets obtained from a knowledge of the system to be controlled, and the complete rules for mapping of these sets. A conclusion fuzzy set can be generated using different theories

The Mamdani-method [2] and other methods based on similar principles generate the conclusion basicly by a weighted considering of the consequent sets chosen by the system of rules from the knowledge. The weighting can be given as the intersection of the fuzzy sets (given on input universe set) and the observation. In this case, as a necessary condition, the knowledge must contain a fuzzy set whose intersection with the observation can not be empty. Or, stated in other way, the conclusion is generable if our knowledge up to this point contains the observation. If this condition fails the conclusion in not generable. As a matter of course, in this way we need the complete knowledge that contains the possible observations. So the support of the union of the input terms and the input universe itself must be equal. In the practice, even stricter conditions apply: the input terms should be located densely. This means that the union of the $\alpha-$ cuts of the input fuzzy sets (e.g. for $\alpha = 0.5$) should be equal with the universe set. The result of this "rule of thumb" is that even in case of a small system, the number of rules is increasing considerably. Having a large number of rules arises a lot of problems both in respect to calculation time and to storage space [3].

The theories are the Fuzzy Interpolation methods [3], [4] and [5]. The importance and the difference to Mamdani-method of the fuzzy rule interpolation is founded upon the ability of the method to deduce from, not only to interconnect the existing knowledge. That is, it is not necesary to have in the knowledge (up to this point) a fuzzy set whose intersection with the observation is not empty. Therefore these methods can be fruitfully used in applications where the available amount of knowledge is limited. In this case the available knowledge consists of the antecedent fuzzy sets $A_{1..m}$ and consequent fuzzy sets $B_{1..m}$ defined on the input universe set X and the output universe set Y respectively, where the support of the union of the sets $A_{1..m}$ is a subset of X. That is the knowledge is incomplete. Of course the rules of relation between antecedent set A_j and consequent set B_j are contained in the knowledge.

So, knowing antecedents $A_{1..m}$ on X, and consequent $B_{1..m}$ on Y, relations $B_j = F^F_j\ (A_j)$ (j=1..m) are already known as fuzzy sets (here F^F denotes a relation. It is not a mathematical function, but a mapping from input (antecedent) fuzzy sets to consequent fuzzy sets). In case of observation A' clonclusion B' can be deduced by known relations between sets A_j and A_{j+1} sorrounding A' and their known consequents B_j and B_{j+1}. Therefore relation $B' = F^F(A')$ can be given by some weighted combination of relations F^F_j and F^F_{j+1}. So these methods give conclusion $B' = B_j$ for different fuzzy sets $A' = A_j$ and generate conclusions for arbitrary observations between A_j and A_{j+1}.
Hence the main difference from the Mamdani-method is that interpolation methods generate the conclusion by means of weighted considering of F relations between antecedent and consequent sets, not by some kind of weighting of consequent fuzzy sets. Making possible in this manner the deduction of the conclusion on the base of limited information.

Multidimensional extension of some interpolation methods exist. In this article we show a multidimensional version of Baranyi Gedeon Kóczy interpolation method [5]. Since we use only the terms flanking the observation let us use the following denotations: X_i is the input universe (i = 1..n, n = the number of input universe), $A_{i,1}$ and $A_{i,2}$ are the antecedents defined on X_i, Y is the output universe B_1 and B_2 are the consequents defined on Y, A'_i is the observed input fuzzy set on X_i and B' is the conclusion which is generated knowing $A'_{1..n}$, $A_{1..n,1}$, $A_{1..n,2}$, B_1 and B_2. The former interpolation methods induce several problems [6]:
- They can be applied only for convex and normal sets.
- They are not even interpetable for arbitrary convex and normal observation fuzzy sets, namely, ordering must hold ($A_{i,j} \prec A'_i \prec A_{i,j+1}$, $B_j \prec B_{j+1}$ where the observation set is A'_i)
- The method does not give a directly interpretable conclusion fuzzy set in every case ("loops" in the membership functions).
- Using trapezoidalal, triangular or crisp sets, the shape is not preserved for the conclusion. It means that in case of fuzzy triangular or trapeziodal terms calculation by the three or four characteristic points e.g. by linear interpolation is not always sufficient as it gives only a rough approximation (except if some rather strict conditions apply). This is an important problem because of the computational complexity aspect.

In this article we will present a new method that can be applied on arbitrary shaped fuzzy terms and that always results in directly "acceptable" sets, further on eliminate all the mentioned problems. To show the essential new points in this method, we will classify the former interpolation methods by their key idea:

First class contains single term deduction methods. The conclusion (B') is generated from observation A' and A_1 a single rule, B_1 using some kind of General Modus Ponens (GMP see e.g. the Revision Principle [7]). These methods can not give a conclusion if the intersection of A' and A_1 is empty. The problem is that in this case the distance between A' and A_1 is not meaningful.
Methods in the second class, use at least two rules. The Mamdani-method and other reasoning methods alike use the degree of matching between observation and at least two antecedents by calculating a weighted average (see e.g.)[2]. This is a natural way of interpolation.

The third class applies approximation for the α–cuts and this can be used even if there is no formal matching. The linear interpolation method is the prototype of the methods in this class. These methods generate conclusion B' to observation A' using at least two rules ($A_1 \rightarrow B_1$, $A_2 \rightarrow B_2$) [4,5,6]. The basic idea of these methods is the following: If given are sequence of observations A_1, A', A_2 and a corresponding of sequence of conclusions B_1, B', B_2 where B' is unknown, B' is found by considering A'-s relative location in X and determining B' from B_1 and B_2 accordingly. If A' is not comparable with A_i ($A_1 \prec A' \prec A_2$) then this method can not be applied. However generalized interpolation and approximation methods (where polynomial or rational functions are used on the characteristic points of the α-cuts, eliminate the difficulty of orderedness condition [8].

The method introduced in this article, goes back to the basic interpolation of fuzzy rules and searches for a conclusion similarly to the way of human thinking. When we hear a new question, at first we summarize our knowledge that is closest to the topic of the question, and we try to find questions that approach the new question as close as possible and whose answers are known. Then based on the comparison of our factual knowledge and the new question we deduce an approximate answer. Following this method, at first, we look for a fitting A_i'' in the sequence of observation "between" known $A_{i,1}$ and $A_{i,2}$, which is the closest to A_i' and we determinate the corresponding conclusion B'' in the sequence of conclusions "between" B_1 and B_2. Actually the first step gives the multidimensional fuzzy set A'' nearest to a multidimensional fuzzy set A'. Because the output base set is one-dimensional we have to transform the multidimensional fuzzy sets A' and A'' into the one-dimensional sets S' and S'' with the same propierties. Then, the conclusion B' to A_i' will be found by evaluating S', S'' and B'' according to the Baranyi Gedeon Kóczy one-dimensional interpolation method [5].

Of course, if we use general type fuzzy terms, the operations with these sets will need much computational effort, because enough points of the sets should be taken into consideration to have a good enough approximation. Therefore, for practical applications we prepare a special method that can be applied for crisp, triangular and trapezoidal fuzzy sets (that can be described by 2,3 and 4 characteristic points resp.).

2 DEFINITIONS

2.1 x_2=F(x_1,p_1,p_2,c_1,c_2) IS THE REVISION FUNCTION

Let x_2= F(x_1,p_1,c_1,c_2) be that function, which results x_2 in such a way that a/b=c/d is true both if $x_1 \le p_1$ and $p_1 < x_1$ (see figure 1).

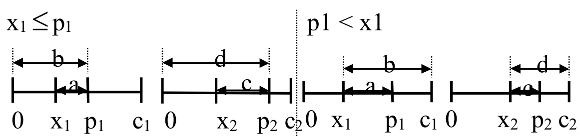


Figure 1

$$x_2=F(x_1, p_1, p_2, c_1, c_2) = \begin{cases} p_2; & \text{if } x_1 = p_1 \\ p_2 - a\dfrac{b\cdot c_2 - p_2}{b\cdot c_1 - p_1}; & \text{otherwise} \end{cases}$$

$$\text{where: } \begin{aligned} a &= p_1 - x_1; \\ b &= \frac{1}{2}\left(1 - \operatorname{sgn}(a)\right); \end{aligned} \qquad (1)$$

2.2 cp(A) CENTRAL POINT OF FUZZY SET A

A ($\langle \forall x \in X,\ \mu_A(x)\rangle$) fuzzy set is given on base set X. The center of the fuzzy set is:

$$cp(A) = \frac{\sup(A_\alpha) + \inf(A_\alpha)}{2}; \text{ where } \alpha = \text{height}(A): \quad (2)$$

(This is a generalization of the concept of the centre of the core.)

3. suppnorm$_{SU}^{SL}$(A) IS THE NORMALIZATION OF THE SUPPORT OF FUZZY SET A FOR GIVEN SU, SL.

Let A ($\langle \forall x \in X,\ \mu_A(x)\rangle$) fuzzy set be given on base set X, then let suppnorm$_{SU}^{SL}$(A) be such a fuzzy set SNA, whose support's minimum value is SL and maximum value is SU. Let the membership function of SNA= suppnorm$_{SU}^{SL}$(A) be:

$$\mu_{SNA}(x) = \mu_A\left(\left(x - cp(A)\right)\cdot\left(\frac{a - cp(A)}{b - cp(A)}\right) + cp(A)\right) \quad (3)$$

Where: SU<cp(A)<SL;
If x<cp(A) then: a=$supp_L$(A)=inf(supp(A)); b = SL;
If x>cp(A) then: a=$supp_U$(A)=sup(supp(A)); b = SU;

3 GENERAL METHOD

According to the introduction, the determination of the concluson fuzzy set can be divided into three main steps. First, the fuzzy set A"$_i$ is to be determined on every input universe X_i. Second, the sets A'$_i$ and A"$_i$ are to be transformed into the sets S' and S" on the universe S. Third, conclusion B' can be determined using S', S" and B".

For determining A"$_i$ and B" there are more conditions to be satisfied: A"$_i$ should be as close to A'$_i$ as possible. The closer is A"$_i$ to $A_{i,j}$ the more similar they are. The same "similarity relation" is required between B" and B_j. In an extreme case, when observation A'$_i$ is identical with $A_{i,j}$ (an already known antcedent) then A"$_i$ the closest information to A'$_i$ should be also identical with $A_{i,j}$. Similarly, B" should be identical with B_j, which implies that conclusions B' and B" are also the same. Therefore in the extreme case, when the observation is $A_{i,j}$, the conclusion should be B_j.

Let us define the crisp distance between two fuzzy sets with the distance of their centres [4]:

$$d (A_1, A_2) = d (cp(A_1), cp(A_2)). \quad (4)$$

To avoid the problem of abnormal membership function shapes for B' no other distance will be calculated, and all points of the membership function will be generated by this distance as a reference. For simplicity, let us consider X_i and Y be normalized for the interval [0,100] i.e.
Mx_i=max(supp(X_i))=My=max(supp(Y))=
Ms=max(supp(S))=100,
mx_i=min(supp(X_i))=my=min(supp(Y))=
ms=min(supp(S))=0.
Let us turn fuzzy sets $A_{i,1}$ and $A_{i,2}$ around their centers as it is shown in figure 2. The rotated curves ($A_{i,1}$ and $A_{i,2}$) are considered as the cross-sections of a geometric solid. Fuzzy set A_i" can be found between $A_{i,1}$ and $A_{i,2}$ as the cross section of this imaginary geometric solid. To get A_i", the closest fuzzy set to A_i', we have to cut the solid at

the position of A_i', using the above introduced distance measure.

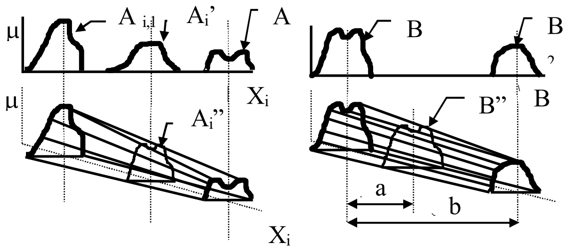


Figure 2: Determination of A_i" and B" by geometric solid cutting.

Turning back the cross-section into its original position we will obtain A_i", a set that satisfies our conditions, namely that is equal to $A_{i,1}$ or $A_{i,2}$ in an extreme case.

There are more possibilities to define the solid based on the rotated fuzzy sets. Therefore, it is possible to handle more than two sets A_j as one solid. In a simple case, when we consider only two fuzzy sets, the geometric solid based on $A_{i,1}$ and $A_{i,2}$ can be easily constructed as a linear translation surface with A_1 and A_2 as rule curves.

The second step is determination of sets S' and S" using $A'_{1..n}$ and $A''_{1..n}$. This transformation can be divided into four parts:

1. Determination of centers and supports of fuzzy sets S' and S".

$$cp(S') = \frac{Ms}{n}\sum_{i=1}^{n}\frac{cp(A'_i)}{Mx_i}; \quad cp(S'') = \frac{Ms}{n}\sum_{i=1}^{n}\frac{cp(A''_i)}{Mx_i}; \quad (5)$$

$$supp_L(S') = \frac{Ms}{n}\sum_{i=1}^{n}\frac{supp_L(A'_i)}{Mx_i}; \quad (6)$$

$$supp_U(S') = \frac{Ms}{n}\sum_{i=1}^{n}\frac{supp_U(A'_i)}{Mx_i}; \quad (7)$$

$$supp_L(S'') = \frac{Ms}{n}\sum_{i=1}^{n}\frac{supp_L(A''_i)}{Mx_i}; \quad (8)$$

$$supp_U(S'') = \frac{Ms}{n}\sum_{i=1}^{n}\frac{supp_U(A''_i)}{Mx_i}; \quad (9)$$

2. Normalization of fuzzy sets A'_i and A''_i to the same supports (SNA'_i and SNA''_i), while their centers does not change. The size of the common support is arbitrary, in this case it is d.

SL = cp(A'_i) - d/2; SU = cp(A'_i) + d/2;

$$SNA'_i = suppnorm_{SU}^{SL}(A'_i); \quad (10)$$

SL =cp(A''_i) - d/2; SU =cp(A''_i) + d/2;

$$SNA''_i = suppnorm_{SU}^{SL}(A''_i); \quad (11)$$

where: i = 1..n;

3. Determination of fuzzy sets SNS' and SNS" point by point, using sets SNA' and SNA". $y \in [0,d]$; Sets A'_i and A''_i could be weighted, depending on needs, by w'_i and w''_i (w'_i, w''_i. $\in [0,1]$).

$$\mu_{SNS'}(cp(S') - d/2 + y) = \frac{1}{n}\sum_{i=1}^{n} w'_i \cdot \mu_{SNA'}(cp(A'_i) - d/2 + y); \quad (12)$$

$$\mu_{SNS''}(cp(S'') - d/2 + y) = \frac{1}{n}\sum_{i=1}^{n} w''_i \cdot \mu_{SNA''}(cp(A''_i) - d/2 + y); \quad (13)$$

4. Determination of fuzzy sets S' and S".

SL=$supp_L$(S'); SU=$supp_U$(S');

$$S' = suppnorm_{SU}^{SL}(SNS'); \quad (14)$$

SL=$supp_L$(S"); SU=$supp_U$(S");

$$S'' = suppnorm_{SU}^{SL}(SNS''); \quad (15)$$

Determination of B" is similar to determination of A''_i fuzzy sets. Satisfying the conditions, the geometrical solid, which is created by turning B_1 and B_2 (fig. 2), should be cut in such way that a/b is equal to:

$$a/b = \frac{\sum_{i=1}^{n}\frac{cp(A'_i) - cp(A_{i,1})}{Mx_i}}{\sum_{i=1}^{n}\frac{cp(A_{i,2}) - cp(A_{i,1})}{Mx_i}}; \quad (16)$$

Determination of B' based on S', S" and B" sets.
Determination of SNB".

SL=cp(B")-d/2; SU=cp(B")+d/2;

$$SNB'' = suppnorm_{SU}^{SL}(B''); \quad (17)$$

Determination SNB' point by point using SNS', SNS", SNB". and the revision function F.

$$x_1 = \mu_{SNA'}(supp_L(SNA') + y); \quad (18)$$

$$p_1 = \mu_{SNA''}(supp_L(SNA'') + y); \quad (19)$$

$$p_2 = \mu_{SNB''}(supp_L(SNB'') + y); \quad (20)$$

$c_1 = 1$; $c_2 = 1$; $y \in [0,d]$;

$$\mu_{SNB'}\left(\mathrm{supp}_L\left(SNB'\right)+y\right)=F\left(x_1,p_1,p_2,c_1,c_2\right); \quad (21)$$

Determination the conclusion B' by suppnorm using set SNB".

$SL=\mathrm{supp}_L(B')$; $SU=\mathrm{supp}_U(B')$; (22)

The conclusion is: $B'=\mathrm{suppnorm}_{SU}^{SL}\left(SNB'\right)$; (23)

The determination of S' and S" by suppnorm using SNS' and SNS" is not neccesary step in the method. It could be over stepped.
The conclusion is a regular fuzzy set in every case, since the normalization of the support must always result into a fuzzy set. Using this method we can get a conclusion for any type of fuzzy sets.

4 SPECIALISED METHOD

This method is a simplified version of the general method, which does not calculate all the points of the membership function, but only the four characteristic points. So this method is applicable for crisp, triangular and trapezoidal fuzzy sets, while it requires only a small amount of computational time.

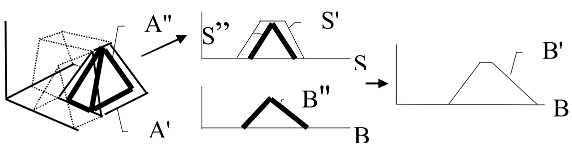


Figure 3: Key-steps in specialised method.

According to the expressed in the introduction, we can consider the observation and the conclusion formed by the multidimensional A' set and the one-dimensional B' fuzzy set respectively, while the knowledge is composed of fuzzy sets A_1, A_2 (multidimensional) and B_1 and B_2 (one-dimensional). So, as mentioned earlier, first we have to determine the fuzzy sets A" and B" (fig. 3). On the base of the sets $A_{i,1}$ and $A_{i,2}$ determined on every input base set the cross sections $A"_i$ must be found.
For simplicity let us denote: $A_{i,3} = A'_i$, $A_{i,4} = A"_i$, $B_3 = B'$, $B_4 = B"$, $S_3 = S'$, $S_4 = S"$ and $a_{i,j,k}$ the k-th characteristic point of $A_{i,j}$ in α, and $d_{A,i,j,k}$ is the distance from $cp(A_{i,j})$. The method can be divided into three parts.

1. Determination of fuzzy sets $A"_i$:
where: $i = 1..n$; $k = 1..4$; $j = 3,4$;

$$d_{A,i,4,k}= d_{A,i,1,k} + (d_{A,i,2,k} - d_{A,i,1,k}) \cdot C_i; \quad (24)$$

$$\text{where: } C_i=\frac{cp(A_{i,3})-cp(A_{i,1})}{cp(A_{i,2}-cp(A_{i,1})};\ S_{j,k}=\frac{Ms}{n}\sum_{i=1}^{n}\frac{a_{i,j,k}}{Mx_i}; \quad (25)$$

2. Determination of the conclusion fuzzy set B':
where: j=1..3;

$$d_{B,4,k} = d_{B,1,k} + (d_{B,2,k} - d_{B,1,k}) \cdot C; \quad (26)$$

$$cp(B_3) = cp(B_4) = cp(B_1) + (cp(B_2) - cp(B_1)) \cdot C; \quad (27)$$

$$\text{where: } C=\frac{cp(S_3)-cp(S_1)}{cp(S_2)-cp(S_1)};\ cp\left(S_j\right)=\frac{Ms}{n}\sum_{i=1}^{n}\frac{cp(A_{i,j})}{Mx_i}; \quad (28)$$

3. Determination of B_3 from A_3, A_4, B_4.
where $Ms' \geq Ms$; $My' \geq My$;
$b_{3,2}=F(s_{3,2};s_{4,2};b_{4,2};cp(S_3);cp(B_3))$ (29)

$b_{3,3}=cp(B_3)+$
$F(d_{S,3,3};d_{S,4,3};d_{B,4,3};Mx-cp(S_3);My-cp(B_3))$; (30)

$b_{3,1} = b_{3,2} \cdot F([s_{3,1} / s_{3,2}]; [s_{4,1} / s_{4,2}]; [b_{4,1} / b_{4,2}]; 1; 1)$;(31)

$b_{3,4}=My'-(My'-b_{3,3}) \cdot F([(Ms'-s_{3,4})/(Ms'-s_{3,3})];$
$[(Ms'-s_{4,4})/(Ms'-_{4,3})];[(My'-b_{4,4})/(My'-b_{4,3})];1;1)$; (32)

5 EXAMPLES

5.1 EXAMPLES OF THE GENERAL METHOD

In the figure 4. the results obtained with the general method in the two-dimensional case can be seen. There are three diagrams in each one of the figures (a,b,c,d). The first and second diagrams represent the input universe X_1 and X_2, while the third one shows the output universe Y. The computer simulation made possible the utilization of fuzzy sets drawn by hand, permitting observation of all the particularities in the process of conclusion generation in any type of fuzzy sets (fig. 4. a,b,c). The fig . c) shows a case, when the conclusion set can be difficultly given using only human comprehension. The fig. d) shows an example of the theory in a extreme case, with crisp sets. The response shows convincingly that B' is the relation AND between sets A'_1 and A'_2 as obtained by Zadeh's max-min operation. And the same is true for the relations between A_{11}, A_{21} and B_1 and A_{12}, A_{22} and B_2. The limits of the conclusion crisp set B' corresponds with the result of the min-max operation with minimal divergence.

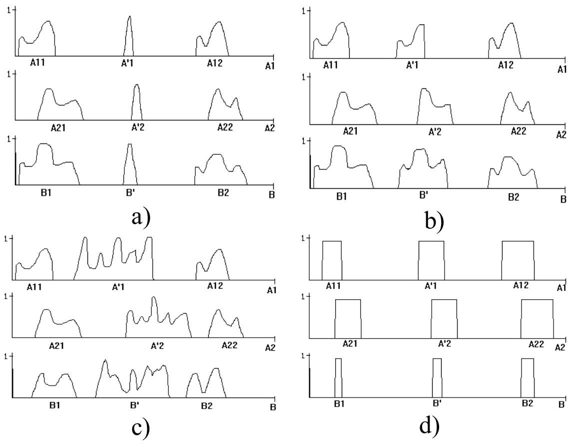
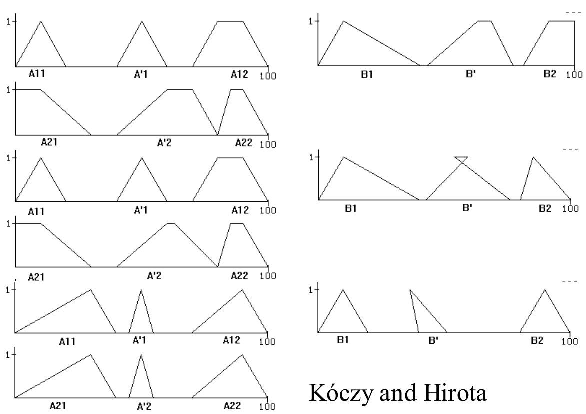


Figure 4: Results of general mehod

5.2 EXAMPLES OF THE SPECIALISED METHOD

In the fig. 5 the results obtained with the specialised method in the two-dimensional case can be seen.



Kóczy and Hirota

Figure 5

The first column shows the input base sets and the observation fuzzy sets A'_1, A'_2, and the antecedent sets A_{11}, A_{12}, A_{21}, A_{22}. In the second column can be found the conclusion fuzzy sets B' generated by Kóczy and Hirota interpolation. The third column contains the results of the Vas-Kalmár and Kóczy-method, and the fourth column the results of specialised method. In the first line every method resulted into a directly interpretable conclusion. Comparing these three different methods it can be said that the Kóczy and Hirota's method usually gives an almost identical conclusion (if it is directly interpretable) with the newly introduced specialized method. The second and third lines present examples where the specialized method still gives directly interpretable conclusions while the others do not.

6 CONCLUSION

The interpolation methods in the previous literature can be applied only on convex sets, while the generalized method introduced in this paper is applicable for arbitrary type fuzzy sets. As a matter of course, the general method makes calculation necessary for "every point" of the set. The specialized method can be applied on the most commonly used crisp, triangular and trapezoidal fuzzy sets, and requires only small computational time. The original linear and nonlinear interpolation methods present another problem, namely the conclusion set is not always convex, therefore it is not sufficient to calculate only the four characteristic points of the set. Our specialized method eliminates this problem, because it results always into crisp, triangular or trapezoidal conclusions, therefore it is enough to calculate only the four characteristic points. Another difference is that the new method offers a directly interpretable conclusion in every case.

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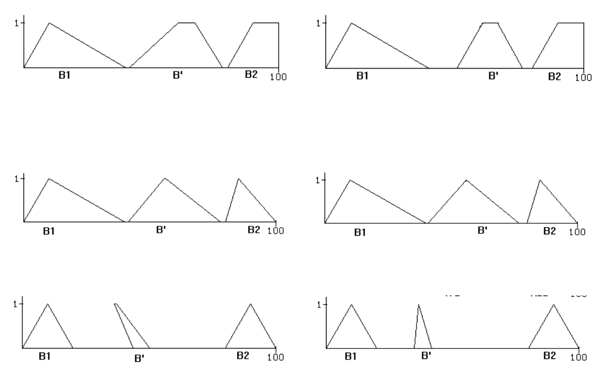

Vas-Kalmár and Kóczy Specialised method

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